Distribution of Dark Matter around a Central Body, Pioneer Effect and Fifth Force

E.Schmutzer*, Jena, Germany
Friedrich Schiller University

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Abstract

Within the framework of the Projective Unified Field Theory the distribution of a dark matter gas around a central body is calculated. As a result the well-known formulas of the Newtonian gravitational interaction are altered. This dark matter effect leads to an additional radial force (towards the center) in the equation of motion of a test body, being used for the explanation of the so-called "Pioneer effect", measured in the solar system, but without a convincing theoretical basis up to now. Further the relationship of the occurring new force to the so-called "fifth force" is discussed.

KEY WORDS: dark matter around center, Pioneer effect, 5th force.

1 Idea of dark matter accretion around a center

Application of the author's 5-dimensional Projective Unified Field Theory (PUFT), being published in a series of papers (Schmutzer 1995a, 1995b, 1999, 2000a, 2001), to a closed homogeneous isotropic cosmological model for the whole time scale (big start to presence) led to following numerical results (Schmutzer 2000b, 2000c), where the index p means present and y year:

$$\begin{aligned} &\text{a)} \quad t_p = 18 \cdot 10^9 \text{ y} && \text{(age of the cosmos),} \\ &\text{b)} \quad H_p = 77.6 \, \frac{\text{km}}{sMpc} && \text{(Hubble factor),} \\ &\text{c)} \quad \mu_p = 3.3 \cdot 10^{-27} \text{ g cm}^{-3} && \text{(mass density).} \end{aligned} \tag{1}$$

Empirical astrophysical estimates of the visible mass density of the normal (mostly baryonic) matter come to the maximum value $\approx 10^{-30}$ g cm^{-3}. We interpreted our result with respect to this difference in the mass densities as a hint at the existence of a dominating dark matter part with a mass density more than two orders of magnitude greater than that of the normal matter.

Our hypothetical model of the cosmos investigated, roughly corresponds to the following picture of the present cosmos:
Our cosmos consists of a gas of dark matter particles (dm-particles) penetrating all matter of the cosmos, particularly also the existing compact objects (stars, nuclei of galaxies etc.) which look like buoys in the dark matter sea. Assuming a homogeneous gas of one sort of particles, by some hypothetical arguments we recently were led to the following numerical values (Schmutzer 2000b):

$$\begin{aligned} &\text{a)} \quad m_p = 2 \cdot 10^{-36} \text{ g} && \text{(rest mass of the dm-particle),} \\ &\text{b)} \quad T_p = 1.78 \text{ K} && \text{(kinetic temperature of the dm-gas).} \end{aligned} \tag{2}$$

*E-mail: eschmu@aol.com

As a result of the attractive interaction of the dm-particles with the compact object considered we expect a statistical distribution of the dm-particles within and around the compact object being embedded in this dm-gas. Let us introduce the notion "accretion cloud" (Umwolke) for this part of the dm-gas which by accretion exceeds the cosmological average distribution of the dm-gas. For simplicity we model the compact object by a homogeneous mass sphere. Further for simplicity we apply the Boltzmann-Maxwell statistics describing spinless particles without degeneracy.

Let us here mention that in a different context and on a different theoretical basis H. Dehnen et al. (Dehnen 1995) used the Bose-Einstein statistics (preferred by physical arguments) and the Fermi-Dirac statistics (dropped) in treating the dark matter in galaxies in order to find a theoretical concept in understanding the problem of the "flat rotation curves" of stars in galaxies. In interesting papers detailled quantum-statistical calculations were performed. We think that for a rough understanding of our approach it is legitimate to simplify the situation by referring to the Boltzmann-Maxwell statistics.

2 Basic equations

In the following PUFT is specialized to the nonrelativistic and weak field case, but we take into account the scalaric effects described by the scalaric field function σ. Then the gravitational field equation is given by

$$\Delta\Phi = 4\pi\gamma_N\mu \tag{3}$$

($\Phi(\boldsymbol{r},t)$ local Newtonian gravitational field of the central body, μ mass density of the central body, γ_N Newtonian gravitational constant as a true constant).

The scalaric field may consist of two parts according to

$$\text{a)}\quad \sigma = \sigma_c + s \qquad \text{with} \qquad \text{b)}\quad |s| \ll |\sigma_c|\,, \tag{4}$$

where $\sigma_c(t)$ is the global scalaric cosmological field and $s(\boldsymbol{r},t)$ the local scalaric field of the central body. Then the scalaric field equation reads

$$\Delta\sigma = \Delta s = -\frac{4\pi\gamma_N\mu}{\sigma_c c^2}. \tag{5}$$

In the general case of a test body moving in the external fields Φ and s as well as in eventually existing external electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ (neglecting in all fields the back reaction) the equation of motion reads:

$$M\left(\frac{\mathrm{d}\,\boldsymbol{v}}{\mathrm{d}\,t} + \operatorname{grad}\Phi + \frac{c^2}{\sigma_c}\operatorname{grad}s + \boldsymbol{v}\frac{\mathrm{d}\ln\sigma_c}{\mathrm{d}\,t}\right) = Q\left(\boldsymbol{E} + \frac{\boldsymbol{v}\times\boldsymbol{B}}{c}\right) \tag{6}$$

(M inertial mass, Q electric charge, $\boldsymbol{v}$ velocity of the test body). Let us mention that the last term on the left hand side of this equation of motion leads to "bremsheat production", particularly also in celestial bodies, investigated in detail in previous papers (Schmutzer 2000c, Schmutzer 2001).

In the following we refer to a test body without electric charge, i.e. $Q = 0$. Because of the same mathematical structure of both the equations (3) and (5), in this approximation it is possible to introduce the combined field function

$$\Psi = \Phi\left(1 - \frac{1}{\sigma_c^2}\right) + c^2\ln\sigma_c + \frac{1}{\sigma_c^2}\Phi_c \tag{7}$$

($\Phi_c(t)$ cosmological value of the Newtonian potential). Hence we obtain the common field equation

$$\Delta\Psi = 4\pi\gamma_N\left(1 - \frac{1}{\sigma_c^2}\right)\mu \tag{8}$$

and the equation of motion

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} + \operatorname{grad}\Psi + \boldsymbol{v}\frac{\mathrm{d}\ln\sigma_c}{\mathrm{d}t} = 0. \tag{9}$$

Let us now use following notation:

$$\mu = \mu_n + \mu_{dm} \tag{10}$$

(μ_n mass density of normal matter, μ_{dm} mass density of dark matter). Application of this latter formula to the interior (i) and to the exterior (e) of the spherical central body leads to the equations

$$\text{a)}\quad \mu_i = \mu_{ni} + \mu_{dmi} \qquad \text{and} \qquad \text{b)}\quad \mu_e = \mu_{ne} + \mu_{dme}\,. \tag{11}$$

Next we split the mass densities into both the parts coming from the global cosmological matter (normal matter and md-matter) and from the local matter (central body and accretion cloud):

$$\text{a)}\quad \mu_{ni} = \mu_{ng} + \mu_{ns}, \qquad \text{b)}\quad \mu_{dmi} = \mu_{dmg} + \hat{\mu}_{dmi} \tag{12}$$

and

$$\text{a)}\quad \mu_{ne} = \mu_{ng}\,, \qquad \text{b)}\quad \mu_{dme} = \mu_{dmg} + \hat{\mu}_{dme}\,. \tag{13}$$

Here the indices refer to:

$g \;\rightarrow\;$ *global* (cosmological contribution),
$s \;\rightarrow\;$ *sphere* (central body contribution).

The roof denotes the dm- contribution induced by the central body, i.e. surplus dark matter to be treated statistically.

For our further calculations it is convenient to introduce the abbreviations

$$\begin{aligned}&\text{a)}\quad \widetilde{\mu}_e = \mu_G + \hat{\mu}_{dme} \qquad \text{with} \qquad \text{b)}\quad \mu_G = \mu_{ng} + \mu_{dmg}, \quad \text{i.e.}\\ &\text{c)}\quad \mu_i = \mu_G + \mu_{ns} + \hat{\mu}_{dmi}\,.\end{aligned} \tag{14}$$

Let us remind that our cosmological model led us to following numerical values of the scalaric cosmological world function:

$$\begin{aligned}&\text{a)}\quad \sigma_c(t \approx 1000\ \text{y}) \approx 3.3\,, \qquad \text{b)}\quad \sigma_{cp} = 65.188\,, \quad \text{i.e.}\\ &\text{c)}\quad \sigma_c^2 \gg 1 \qquad \text{(for the time scale under consideration).}\end{aligned} \tag{15}$$

Then the field equation (8) for the interior and exterior reads:

$$\text{a)}\quad \Delta\Psi_i = 4\pi\gamma_N\mu_i, \qquad \text{b)}\quad \Delta\Psi_e = 4\pi\gamma_N\widetilde{\mu}_e. \tag{16}$$

Further from (7) results

$$\Psi = \Phi + c^2\ln\sigma_c\,. \tag{17}$$

Next, analogously to the above formulas for the mass density of the dm-particles we now write down the corresponding formulas for the particle number density n_{dm} being related to the mass density as follows (m mass of a dm-particle):

$$\text{a)}\quad \mu_{dm} = mn_{dm}\,, \qquad \text{b)}\quad \mu_{dmg} = mn_{dmg}\,, \qquad \text{c)}\quad \hat{\mu}_{dm} = m\hat{n}_{dm}\,. \tag{18}$$

We find

$$n_{dm} = n_{dmg} + \hat{n}_{dm}\,. \tag{19}$$

In (18) and (19) we omitted the idices i and e.

Since the time dependence of the cosmological quantities (e.g. of $\sigma_c(t)$) is extremly slow (adiabatic time dependence), compared with the time dependence of the cosmogonic processes of the cosmic objects, we omit explicit writing of the time t in such cosmological quantities. Particularly constants of spatial integration implicitly involve this adiabatic time dependence.

3 Statistical dm-particle distribution in the accretion cloud

3.1 Distribution function

According to our accretion concept presented in Section 1 the statistical dm-particle number density for the case of spherical symmetry reads (k Boltzmann constant):

$$n \equiv \hat{n}_{dm} = \bar{n}\left[\exp\left(-\frac{m\chi(r)}{\mathrm{k}T}\right) - 1\right], \tag{20}$$

where $\bar{n}$ denotes the dm-particle number density far away from the cental body:

$$\text{a)} \quad \bar{n} = n(r = \infty) \qquad \text{with} \qquad \text{b)} \quad \chi_\infty \equiv \chi_e(r = \infty) = 0\,. \tag{21}$$

The potental function χ used is related to Ψ by

$$\text{a)} \quad \Psi = \chi + \Psi_c\,, \qquad \text{where} \qquad \text{b)} \quad \Psi_c = \Phi_c + c^2 \ln \sigma_c\,. \tag{22}$$

Using (20) we can write the equations (16) as

$$\begin{aligned} \text{a)} \quad \Delta\chi_i &= 4\pi\gamma_N\left[\mu_0 + m\bar{n}\left\{\exp(-\tfrac{m\chi_i}{\mathrm{k}T}) - 1\right\}\right], \\ \text{b)} \quad \Delta\chi_e &= 4\pi\gamma_N\left[\mu_G + m\bar{n}\left\{\exp(-\tfrac{m\chi_e}{\mathrm{k}T}) - 1\right\}\right], \end{aligned} \tag{23}$$

where the quantity

$$\mu_0 = \mu_G + \mu_{ns} = const \tag{24}$$

(referring to a homogeneous sphere) means the mass density of the sphere.

Let us for the following use the approximation assumption

$$\left|\frac{m\chi}{\mathrm{k}T}\right| \ll 1 \tag{25}$$

being well fulfilled in our further applications. Hence series expansion of (20) leads to the linearity

$$n = -\frac{\bar{n}m\chi}{\mathrm{k}T} \tag{26}$$

between n and χ. This simplification means that we are able to perform the further calculations analytically. Then the equations (23) commonly treated (omitting the indices i and e) read

$$\Delta\chi + \kappa^2\chi = 4\pi\gamma_N\hat{\mu}\,, \tag{27}$$

where $\hat{\mu}$ means μ_0 for the interior or μ_G for the exterior, and κ^2 is defined by

$$\kappa^2 = \frac{4\pi\gamma_N m^2\bar{n}}{\mathrm{k}T}\,. \tag{28}$$

Let us here mention that the author's dissertation had referred to the theory of strong electrolytes, where according to the Debye-Milner theory the statistical treatment of a (negative) ion cloud around a fixed (positive) ion (and vice versa) plays an important role. This knowledge induced the author's idea to treat the dark matter gas analogously. But in the first moment he was surprised that, though between positive and negative electric charges as well as between masses attractive forces act, in his basic equation (27) the second term on the left hand side appeared with a positive sign, in contrast to the theory of strong electrolytes. The consequence of this fact is that in the Debye-Milner theory a Yukawa-like term with exponential

decrease occurs, whereas in this theory an interesting phenomenon of periodicity is met.

In solving the equation (27) we first go over to spherical polar coordinates:

$$\frac{\mathrm{d}}{\mathrm{d}\,r}\left(r^2\frac{\mathrm{d}\,\chi}{\mathrm{d}\,r}\right)+\kappa^2r^2\chi=4\pi\gamma_N\hat{\mu}r^2\,. \tag{29}$$

3.2 Interior

In this case the substitution $\hat{\mu}\to\mu_0$ has to be applied. Then equation (29) reads

$$\frac{\mathrm{d}}{\mathrm{d}\,r}\left(r^2\frac{\mathrm{d}\,\chi_i}{\mathrm{d}\,r}\right)+\kappa^2r^2\chi_i=4\pi\gamma_N\mu r^2\,. \tag{30}$$

The solution of this inhomogenious differential equation is

$$\chi_i=\frac{B_1}{r}\cos(\kappa r)+\frac{B_2}{r}\sin(\kappa r)+B_0 \tag{31}$$

with

$$\text{a)}\quad B_0=\frac{\mu_0\mathrm{k}T}{m^2\bar{n}}=\frac{3\gamma_N M_c}{\kappa^2r_0^3}\,,\qquad\text{where}\qquad\text{b)}\quad M_c=\frac{4\pi\mu_0r_0^3}{3} \tag{32}$$

is the mass of the central body with radius r_0 ($B_1$and B_2 constants of integration). Regularity at $r=0$ leads to $B_1=0$, i.e.

$$\chi_i=\frac{B_2}{r}\sin(\kappa r)+B_0\,. \tag{33}$$

Differentiaton gives

$$\frac{\mathrm{d}\,\chi_i}{\mathrm{d}\,r}=\frac{B_2}{r^2}\{\kappa r\cos(\kappa r)-\sin(\kappa r)\}\,. \tag{34}$$

3.3 Exterior

This case differs from the previous one by the substitution $\hat{\mu}\to\mu_G$. Analogously we receive the solution

$$\chi_e=\frac{A_1}{r}\cos(\kappa r)+\frac{A_2}{r}\sin(\kappa r)+A_0 \tag{35}$$

with

$$A_0=\frac{\mu_G\mathrm{k}T}{m^2\bar{n}} \tag{36}$$

(A_1 and A_2 constants of integration). For physical reasons this formally correct result has to be changed, because it is not in accordance with the boundary condition (21b): We know that the global source μ_G has already be taken into account by solving the cosmological differential equations. Therefore in this local context we have to drop the inhomogenious contribution by setting $A_0=0$, i.e.

$$\chi_e=\frac{A_1}{r}\cos(\kappa r)+\frac{A_2}{r}\sin(\kappa r)\,. \tag{37}$$

Differentiation leads to

$$\frac{\mathrm{d}\,\chi_e}{\mathrm{d}\,r}=\left(A_2\kappa-\frac{A_1}{r}\right)\frac{\cos(\kappa r)}{r}-\left(A_1\kappa+\frac{A_2}{r}\right)\frac{\sin(\kappa r)}{r}\,. \tag{38}$$

In performing these calculations we realized that following basic functions play an important role:

$$\text{a)}\quad U(z)=z\sin z+\cos z\qquad\text{and}\qquad\text{b)}\quad V(z)=\sin z-z\cos z\,. \tag{39}$$

Figures 1 and 2 show the plots of the functions $U(z)$ and $V(z)$.

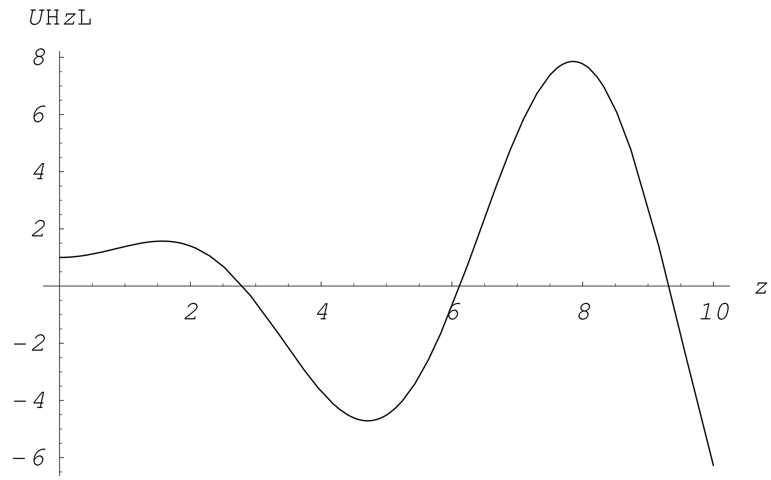

Figure 1: Dark matter basic function U

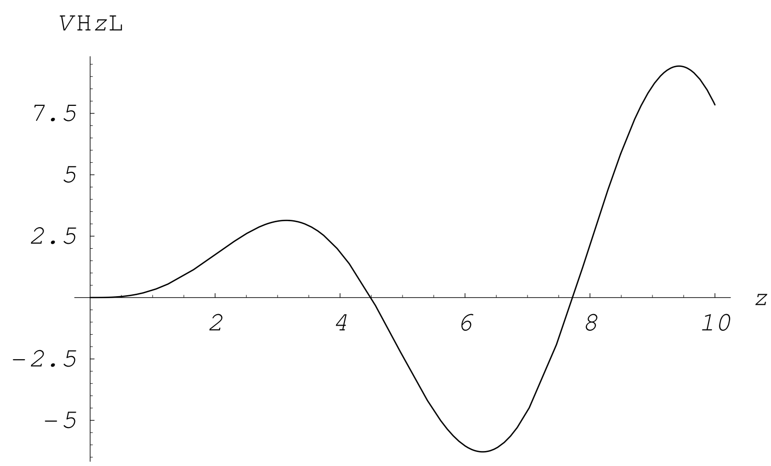

Figure 2: Dark matter basic function V

3.4 Continuity conditions and the final form of the potentials and of the dm-particle number densities

At the surface of the spherical central body following continuity conditions have to be fulfilled:

$$\text{a)}\quad \chi_i(r_0) = \chi_e(r_0)\,, \qquad \text{b)}\quad \frac{\mathrm{d}\,\chi_i(r_0)}{\mathrm{d}\,r} = \frac{\mathrm{d}\,\chi_e(r_0)}{\mathrm{d}\,r}\,. \tag{40}$$

If these conditions are satisfied, then because of (20) the analogous conditions for the dm-particle number densities result:

$$\text{a)}\quad n_i(r_0) = n_e(r_0)\,, \qquad \text{b)}\quad \frac{\mathrm{d}\,n_i(r_0)}{\mathrm{d}\,r} = \frac{\mathrm{d}\,n_e(r_0)}{\mathrm{d}\,r}\,, \tag{41}$$

i.e. thus the continuity of the dm-particle number density is guaranteed, too. Looking at (33) and (37) we recognize that we have to cope with the 3 constants of integration B_2 , A_1 and A_2. From both the conditions (40) we obtain the relations

$$\text{a)}\quad A_1 = -\frac{B_0}{\kappa}V(\kappa r_0)\,, \qquad \text{b)}\quad B_2 - A_2 = -\frac{B_0}{\kappa}U(\kappa r_0)\,. \tag{42}$$

For physical reasons it seems to be sensible to accept that the potential in the whole space (interior and exterior) may be determined solely by the central mass. This means setting $A_2 = 0$. Then finally the decisive potentials (33) and (37) including their derivatives (34) and (38) read:

$$\begin{aligned} &\text{a)}\quad \chi_i = B_0\left\{1 - \frac{1}{\kappa r}\,U(\kappa r_0)\sin(\kappa r)\right\}\,, \\ &\text{b)}\quad \frac{\mathrm{d}\,\chi_i}{\mathrm{d}\,r} = \frac{B_0}{\kappa r^2}\,U(\kappa r_0)V(\kappa r)\,; \end{aligned} \tag{43}$$

$$\begin{aligned} &\text{a)}\quad \chi_e = -\frac{B_0}{\kappa r}V(\kappa r_0)\cos(\kappa r)\,, \\ &\text{b)}\quad \frac{\mathrm{d}\,\chi_e}{\mathrm{d}\,r} = \frac{B_0}{\kappa r^2}V(\kappa r_0)U(\kappa r)\,. \end{aligned} \tag{44}$$

Let us mention that in the center of the central body the equations

$$\text{a)}\quad \chi_i(r=0) = B_0\{1 - U(\kappa r_0)\}\,, \qquad \text{b)}\quad \frac{\mathrm{d}\,\chi_i(r=0)}{\mathrm{d}\,r} = 0 \tag{45}$$

are valid.

Furthermore, according to (26) we also present with the help of (43a) and (44a) the final form of the distribution functions n_i and n_e:

$$\begin{aligned} &\text{a)}\quad n_i = -\frac{\bar{n}mB_0}{\mathrm{k}T}\left\{1 - \frac{1}{\kappa r}U(\kappa r_0)\sin(\kappa r)\right\}\,, \\ &\text{b)}\quad n_e = \frac{\bar{n}mB_0}{\mathrm{k}T\kappa r}V(\kappa r_0)\cos(\kappa r)\,. \end{aligned} \tag{46}$$

3.5 Series expansions

For various applications it is convenient to know the first terms of the series expansion of some functions presented above:

$$\begin{aligned} &\text{a)}\quad U(z) = 1 + \frac{1}{2}z^2 - \frac{1}{8}z^4\,, \\ &\text{b)}\quad V(z) = \frac{1}{3}z^3(1 - \frac{1}{10}z^2)\,; \end{aligned} \tag{47}$$

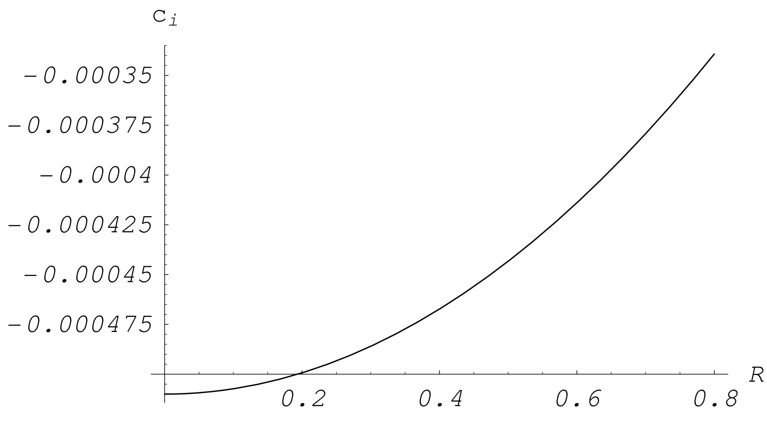

Figure 3: Dependence of the interior potential on radius R

$$\text{a)}\quad \chi_i = -\frac{1}{2}B_0\,(\kappa r_0)^2\left[1-\frac{1}{3}\left(\frac{r}{r_0}\right)^2-\frac{1}{4}(\kappa r_0)^2\left\{1+\frac{2}{3}(\frac{r}{r_0})^2-\frac{1}{15}\left(\frac{r}{r_0}\right)^4\right\}\right]\,,$$

$$\text{b)}\quad \frac{\mathrm{d}\,\chi_i}{\mathrm{d}\,r} = \frac{1}{3}B_0\kappa^2 r\left[1+\frac{1}{2}(\kappa r_0)^2\left\{1-\frac{1}{5}(\frac{r}{r_0})^2\right\}\right]\,; \tag{48}$$

$$\text{a)}\quad \chi_e = -\frac{1}{3r}B_0\kappa^2 r_0^3\left[1-\frac{1}{2}(\kappa r)^2\left\{1+\frac{1}{5}\left(\frac{r_0}{r}\right)^2\right\}\right]\,,$$

$$\text{b)}\quad \frac{\mathrm{d}\,\chi_e}{\mathrm{d}\,r} = \frac{1}{3r^2}B_0\kappa^2 r_0^3\left[1+\frac{1}{2}(\kappa r)^2\left\{1-\frac{1}{5}\left(\frac{r_0}{r}\right)^2\right\}\right]\,; \tag{49}$$

$$\chi_i(r=0) = -\frac{1}{2}B_0(\kappa r_0)^2\left[1-\frac{1}{4}(\kappa r_0)^2\right]\,. \tag{50}$$

One should realize that the special case $\kappa = 0$ of (48a) and (49a) corresponds to usual Newtonian physics with the results

$$\text{a)}\quad \chi_{Ni} = -\frac{1}{2}B_0\,(\kappa r_0)^2\left[1-\frac{1}{3}\left(\frac{r}{r_0}\right)^2\right] = -\frac{3\gamma_N M_c}{2r_0}\left[1-\frac{1}{3}\left(\frac{r}{r_0}\right)^2\right]\,,$$

$$\text{b)}\quad \chi_{Ne} = -\frac{1}{3r}B_0\kappa^2 r_0^3 = -\frac{\gamma_N M_c}{r}\,. \tag{51}$$

Concluding this section, we point to the Figures 3 and 4 which show plots of the radial dependence of the potential χ on r in different ranges (qualitative presentation for $B_0 = 1$). Analogously the Figures 5 and 6 exhibit the radial dependence of the dm-particle number density n on r (again qualitative presentation for $B_0 = 1$). For these Figures the value $\kappa = 0.04[(\mathrm{kpc})^{-1}]$ was chosen which apparently seems to be appropriate for the rotation curves of the Galaxy.

4 Pressure in the central body

For physical reasons the radial course of the pressure is interesting, too. Approaching this aim, here instead starting from the equation of motion of a test body (6), we

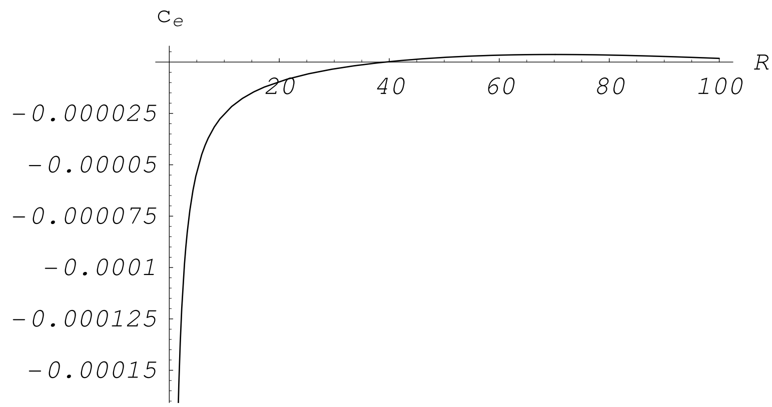

Figure 4: Dependence of the exterior potential on radius R

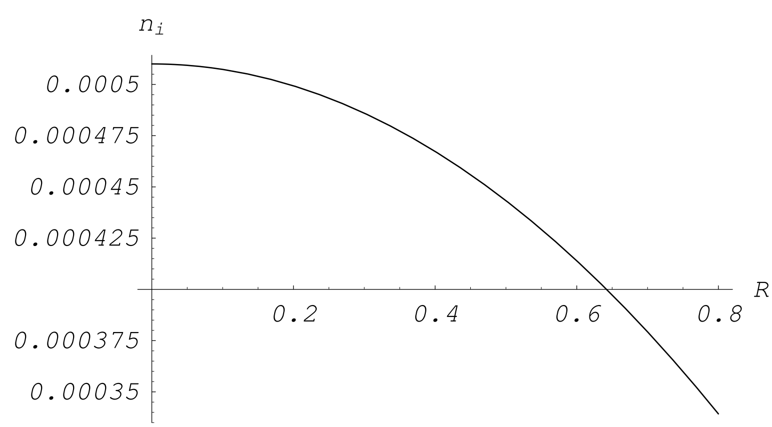

Figure 5: Dependence of the interior dm-particle number density on radius R

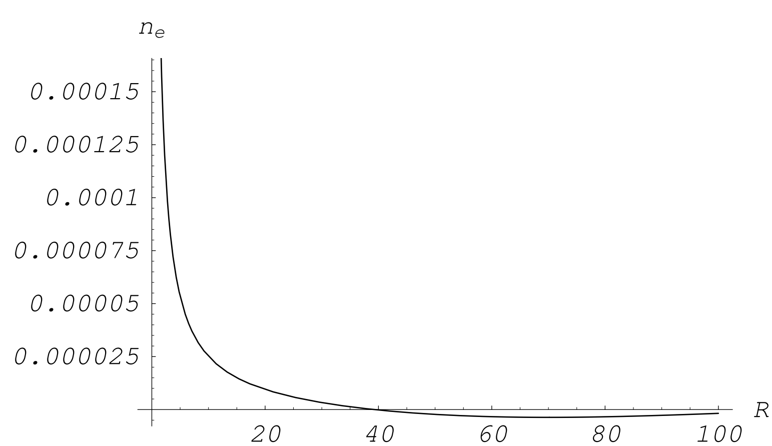

Figure 6: Dependence of the exterior dm-particle number density on radius R

have to start from our generalization of the Navier-Stokes equation of a continuum

$$\mu\left(\frac{\mathrm{d}\,\boldsymbol{v}}{\mathrm{d}\,t}+\operatorname{grad}\Phi+\frac{c^2}{\sigma_c}\operatorname{grad}s+\boldsymbol{v}\frac{\mathrm{d}\ln\sigma_c}{\mathrm{d}\,t}\right)=\rho\left(\boldsymbol{E}+\frac{\boldsymbol{v}\times\boldsymbol{B}}{c}\right)-\operatorname{grad}p- \\ -\eta\left(\Delta\boldsymbol{v}+\frac{1}{3}\operatorname{grad}\operatorname{div}\boldsymbol{v}\right) \tag{52}$$

(p pressure, ρ electric charge density, η viscosity). Specialization to the case $\boldsymbol{v}=0$ (hydrostatics), $\rho=0$ (electro-neutrality), $\eta=0$ (perfect fluid) and mass homogeneity leads to the equation

$$\operatorname{grad}\left(\Phi+\frac{c^2}{\sigma_c}s+\frac{p}{\mu_0}\right)=0 \tag{53}$$

or

$$\text{a)}\quad \operatorname{grad}\left(\chi+\frac{p}{\mu_0}\right)=0\,,\qquad \text{i.e.}\qquad \text{b)}\quad p=-\mu_0\chi+\bar{p} \tag{54}$$

($\bar{p}$ constant of integration). Using (43a) we find

$$p=\mu_0 B_0\left\{\frac{1}{\kappa r}U(\kappa r_0)\sin(\kappa r)-1\right\}+\bar{p}\,. \tag{55}$$

The postulate of vanishing pressure at the surface of the sphere, $p(r=r_0)=0$, leads to the result

$$p=\mu_0 B_0 U(\kappa r_0)\left[\frac{\sin(\kappa r)}{\kappa r}-\frac{\sin(\kappa r_0)}{\kappa r_0}\right] \tag{56}$$

with

$$p(r=0)=\mu_0 B_0 U(\kappa r_0)\left[1-\frac{\sin(\kappa r_0)}{\kappa r_0}\right]\,. \tag{57}$$

Specialization to the Newtonian case $\kappa=0$ gives the result

$$p_N=\frac{\gamma_N M_c\mu_0}{2r_0}\left[1-(\frac{r}{r_0})^2\right]\,. \tag{58}$$

5 Radial drift of a test body ("Pioneer effect")

Recent evaluation of radio metric data from Pioneer 10/11, Galileo and Ulysses spacecraft indicates an anomalous, constant acceleration acting on the spacecraft, being about 50 AU far from the Sun, with a magnitude $\approx 8.5\cdot 10^{-8}\mathrm{cm/s}^2$, directed towards the Sun (Anderson et al. 1998). Apparently the explanation of this new effect seems to be related to a correction of Newtonian mechanics by additional terms. But also general-relativistic explanation approaches are under discussion (Rosales and Sánchez-Gómez 2000). In the following we try to find an explanation of this radial drift effect by applying our theory.

The equation of motion of a test body (9) with the help of (22a) takes the form

$$\frac{\mathrm{d}\,\boldsymbol{v}}{\mathrm{d}\,t}+\operatorname{grad}\chi+\boldsymbol{v}\frac{\mathrm{d}\ln\sigma_c}{\mathrm{d}\,t}=0\,. \tag{59}$$

From this equation we learn that for a spherically symmetric potential, as investigated above, the appearing acceleration A of the body considered can be splitted into the Newtonian part A_N and the scalaric part A_S according to

$$A=\frac{\mathrm{d}\,\chi}{\mathrm{d}\,r}=A_N+A_S\,. \tag{60}$$

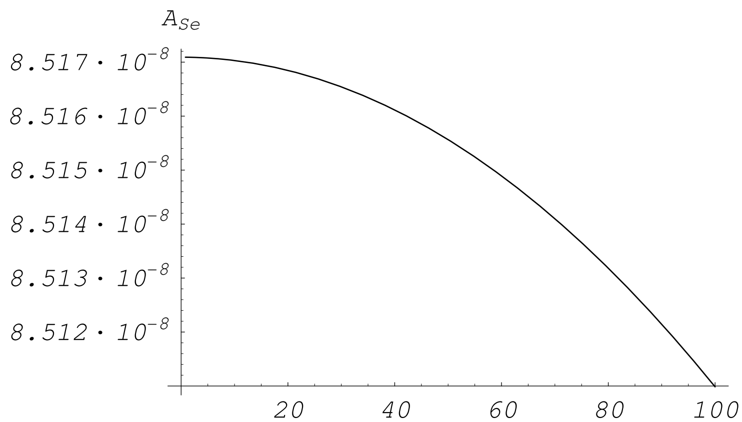

Figure 7: Dependence of the additional radial acceleration on radius R

Referring to the interior and exterior of the central body we identify

$$\text{a)}\quad A_{Ni} = \frac{\gamma_N M_c r}{r_0^3} \qquad \text{and} \qquad \text{b)}\quad A_{Ne} = \frac{\gamma_N M_c}{r^2} \tag{61}$$

(internal and external Newtonian accelerations). With the help of (43b), 48b) and (44b), (49b) the additional scalaric parts take the shape

$$\begin{aligned} &\text{a)}\quad A_{Si} = \frac{\gamma_N M_c \kappa^2}{(\kappa r_0)^3}\left[\frac{3U(\kappa r_0)V(\kappa r)}{(\kappa r)^2} - \kappa r\right] \approx \frac{\gamma_N M_c \kappa^2 r}{2r_0}\left[1 - \frac{1}{5}\left(\frac{r}{r_o}\right)^2\right], \\ &\text{b)}\quad A_{Se} = \frac{\gamma_N M_c}{r^2}\left[\frac{3V(\kappa r_0)U(\kappa r)}{(\kappa r_0)^3} - 1\right] \approx \frac{\gamma_N M_c \kappa^2}{2}\left[1 - \frac{1}{4}(\kappa r)^2\right]. \end{aligned} \tag{62}$$

Remembering that this series expansion is valid for $(\kappa r)^2 \ll 1$, we apply this latter formula to the empirical facts mentioned above:

$$\frac{\gamma_N M_c \kappa^2}{2}\left[1 - \frac{1}{4}(\kappa r)^2\right] = 8.5 \cdot 10^{-8}\text{cm}\,\text{s}^{-2}\,. \tag{63}$$

Inserting the well-known values of the Newtonian gravitational constant $\gamma_N = 6.68 \cdot 10^{-8}\text{g}^{-1}\,\text{cm}^3\text{s}^{-2}$, the mean distance of the spacecraft considered from the Sun $r = 50$ AU, and the mass of the Sun $M_c = 1.989 \cdot 10^{33}\,\text{g}$, we arrive at the value of κ for the Sun (1 AU $= 1.496 \cdot 10^{13}\,\text{cm}$)

$$\kappa = 3.58 \cdot 10^{-17}\,\text{cm}^{-1} = 5.36 \cdot 10^{-4}(\text{AU})^{-1}\,. \tag{64}$$

Though our theory predicts a rather constant value of the radial acceleration far away from the Sun, nevertheless this acceleration value decreases at even greater distance. Fig.7 shows the course of the curve in detail.

6 Dark matter induced new interaction and the "5th force" interaction

Nearly twenty years ago intensive discussions on the validity of the equivalence principle and on an eventually existing new interaction, acting also in macrophysics, took place. Many proposals were published, where mainly the concrete approach of

Fischbach et al. met considerable attention (Fischbach et a. 1986) who introduced the notion “fifth force” with the Yukawa type potential energy between two particles m_1 and m_2 :

$$V(r) = -\frac{G_\infty m_1 m_2}{r}\left[1 + \alpha \exp\left(-\frac{r}{\lambda}\right)\right] \tag{65}$$

(G_∞ gravitational constant at large distance, α and λ free parameters).

According to our external potential (49) the potential energy between such two masses considered reads ($\gamma_N \to G_\infty$)

$$V_S = -\frac{3G_\infty m_1 m_2}{r(\kappa r_0)^3} V(\kappa r_0)\cos(\kappa r) \approx -\frac{G_\infty m_1 m_2}{r}\left[1 - \frac{1}{2}(\kappa r)^2\left\{1 + \frac{1}{5}\left(\frac{r_0}{r}\right)^2\right\}\right] . \tag{66}$$

Since the functional structures of the potential energies (65) and (66) are different, the free parameters α , λ and κ, r_0 don't correspond uniquely. Roughly speaking, only the identification $\frac{\alpha}{\lambda^2} \to -\kappa^2$ is possible.

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Address of the author:

Ernst Schmutzer Cospedaer Grund 57 D-07743 Jena Germany